1972, *Phys. Earth Planet. Interiors* **6**, 182–197. North-Holland Publishing Company, Amsterdam

THE PRESENT THERMAL STATE OF THE TERRESTRIAL PLANETS

D. C. TOZER

The University, Newcastle upon Tyne NE1 7RU, England

It is suggested that the thermal state of bodies can only be properly discussed as an aspect of their internal dynamics. The present paper attempts to do this for bodies of roughly meteoritic composition in the range of sizes of planetary objects. The rheological behaviour of planetary material is assumed to be represented by a Newtonian viscosity varying exponentially with temperature, and only the consistency of this assumption with the elastic view of the planets is considered. It is shown that for the whole range of physically plausible values of the material parameters, the temperature distribution associated with a solution to the problem of internal dynamics for bodies with external radii larger than about 800 km departs significantly from that predicted on conduction theory arguments. For bodies in the range of sizes of the terrestrial planets, the viscosity–temperature relationship plays a dominant role in setting the absolute temperature of the central regions. The magnitude of the velocity of internal motions, the non-hydrostatic stresses and viscosity within the convective zones are relatively insensitive to the choice of material parameter values. These derived quantities are in substantial agreement with values assigned for the Earth on the basis of various observations. The role of phase changes in limiting the radial scale of motions and the case for considering thermal problems as steady state problems is discussed.

1. Introduction

In a recent paper (TOZER, 1972) concerning the thermal state of the Moon, I expressed the opinion that reasonable success in understanding energy transfer as heat in material systems of laboratory and technological size had, over the years, closed our eyes to the approximate nature of certain key ideas used in those applications of the theory. When such approximations were inadvertently applied to the study of really large systems – of which we may regard planets as an interesting observable sample – not only was there the possibility of great numerical error, but also a misdirection of effort to elucidate questions that were not necessarily important in this new domain. For example, an analytical simplification, hallowed only by long experience with small scale problems, was to spawn a single differential equation for temperature, a truncated version of an equation expressing conservation of energy during its transfer, and describe it as governing a separate process (heat conduction) of energy transfer for matter in the 'solid' state. The success of this approximation as a physical theory could be attributed to the fact that simple empirical procedures established a characteristic temperature for a 'small' quantity of most materials and the inequality 'local temperature < characteristic temperature' appeared to serve as a criterion of 'solidity' in the above sense.

Two questions prompt us to reexamine this approximation. The first, entirely theoretical in nature, gives us good a priori grounds for rejecting any such locally defined criterion as a properly formulated condition for truncating a set of coupled partial differential equations (expressing energy, mass and momentum conservation for a continuum) to a single equation. Coupled partial differential equations imply that the value of any one field variable at a particular point, e.g. the velocity vector V, depends on values of the other field variables everywhere. It follows that any trial solution of these equations with $V = 0$ (equivalent to the truncation) can only be sustained by some condition on the associated situation existing throughout the entire system.[1,2] In reexamining this problem, one of our aims must be a better understanding of how a

[1] Symbols are explained at the end.

[2] It is often difficult to find words that do justice to this "global" aspect of transfer problems. Explanations of motion, for example, that involve a conceptual division of the system – this portion pushes/pulls the rest, often beg the question and, in this case, just confuse one's understanding of Newton's third law applied to the internal forces.

locally defined criterion for truncation works as well as it does – at least, for 'small' systems.

The second question, appearing in a particular guise with this 'solidity' criterion, concerns the nature of the restrictions placed on the form of a continuum mechanical theory for a large system by physical data obtained from very small specimens of it. This matter is controversial and I can only outline the factors that have influenced my choice of approach. One's first reaction is to take the principle that the whole is determined by the sum of its parts very literally, and strive to measure the properties of as many handy sized specimens as possible. The utter impossibility of achieving significant coverage, even for planet Earth, is generally recognised, but what is not so obvious is that problems comparable in difficulty to computing the specimen behaviour from atomic properties would be experienced in using such information for planetary models. The crucial points to note here are that individual specimen data are necessarily incomplete as well as imprecise and the form of the conservation equations that result if laboratory observations are interpreted with continuum mechanics does not admit any precise length scaling. To emphasise the magnitude of the problem this makes, consider a set of geometrically similar bodies formed by the aggregation of systems identical to an observed specimen. Then one can expect the response of such 'homogeneous' bodies to the particular intensive boundary conditions that had been used to assign specimen properties to change progressively as the scale factor is increased and to depend on properties of the material that were unassigned or undetectable for the original specimen. It is very easy to misconstrue empirically assigned parameters as values of intensive variables and forget that they owe their very existence to a process that cannot be justified a priori – the choice of a particular continuum mechanical theory.[3]

These difficulties of scaling have convinced me that one has to use a logically equivalent approach for all systems regardless of their size. The aim is to devise the simplest continuum mechanical theory that interprets and predicts selected observable or potentially observable aspects of a system. Just as an interpretation of laboratory observations that uses these theories is not and could never be tested on a point to point basis throughout the system, so one must resist the temptation to demand this of a larger system. Instead of thinking of properties as intensive variables, they only have meaning with respect to a certain body of data and to a certain theory. The remark that we do not know the properties of a system beforehand in order to develop the 'correct' theory of its behaviour is as meaningless for a planet as it is for a laboratory specimen.

2. Choice of material models for the terrestrial planets

Although the above approach sets no limit on the choice of a theory from the innumerable number that are compatible with the general principles of continuum mechanics (see e.g. TRUESDELL and NOLL, 1965), a brake on idle speculation and a natural order in which to investigate these theories is set by two considerations.

The first recognises that confidence in a prediction is increased if one realises that it is insensitive to the possible details of a model, and therefore special significance attaches to those particular continuum mechanical theories that represent a limiting form for a host of special theories. It is as such limiting forms for the 'small' and the 'slow' deformation of materials with fading memory of their past configurations that modern continuum mechanics views linear viscoelastic and viscous continuum theories (TRUESDELL and NOLL, 1965).[4] This also accounts for their widespread utility in experimental science and technology. What is meant by 'small' or 'slow' depends on the level of approximation that is tolerable; but in choosing to investigate the thermal state of the terrestrial planets as an aspect of the theory of their slow deformation, we must remember that seismology, through its assignment of elastic parameters, has demonstrated that planetary

[3] It is just the success of scaling laboratory data by factors utterly trivial to those contemplated in planetary physics e.g. in engineering practice, that has created the belief that material properties are intensive variables. This was forcibly brought home to me when attempting to assess the concept of a finite yield strength for a planet composed of material for which a low stress thermally activated creep process had been postulated.

[4] Of course, both classical elasticity theory and Newtonian viscous theories can be placed under the heading of linear viscoelastic theories, but the explicit reference to a linear (Newtonian) viscous theory conveys the fact that the viscoelasticity of these materials itself has a specific limiting form for slow deformation. I am not aware that a comparatively rigorous proof exists for the Fourier law of heat conduction as a limiting form but suspect that one can be found.

material has a finite memory (in the above sense), and this could reveal itself if the solutions to the Newtonian viscous problem develop particular features. Necessary conditions for the consistency of viscous solutions with seismology are that the viscous strain be very small in a time $\tau = \eta/\mu$ and that τ be much longer than the period of seismic disturbances. The first of these conditions may alternatively be written: viscous stress $\ll \mu$.

The other consideration attempts to give proper weight to the idea of physical plausibility. For the same reasons that the incomplete results of laboratory experiment cannot be translated into relevant planetary data, one cannot argue rigorously in the reverse sense, but clearly the idea of physical plausibility is concerned with the results of attempting to do so. Rather than excluding certain planetary theories, the idea of physical plausibility only suggests a priority for investigation of those models that incorporate laws of material behavior (constitutive relations), which if also applied to systems of laboratory scale, give solutions not in qualitative disagreement with our general empirical knowledge.

With these ideas in mind, let us first examine the slow deformation of planets whose material properties are defined by particular numerical values of the quantities ρ, κ, α, H, C_p and for which the kinematic viscosity depends on temperature through equations of the form:

$$\nu = \nu_0 \, T \, e^{AT_M/T}, \qquad (1)$$

$$\nu_0 = B\,[\tfrac{1}{2}l_1(1+\tanh C\,(T_M - T)+l_2]^2. \qquad (2)$$

We may take this definition of planetary material properties by the eleven numbers ρ, κ, α, H, C_p, A, B, C, T_M, l_1, l_2 as a statement of homogeneity and the absence of pressure effects. The case for dropping either of these assumptions will be considered, but for the moment we are concerned to acquire general insight about solutions of the slow deformation problem for planets with the minimum of analytical complexity.

3. Analysis of models

Our initial task is to use the idea of physical plausibility to identify a region of this material parameter space which has first priority for investigation. Rather than explore the whole of this region, we must be content to select a particular representative point within it and examine how the solutions change as the point defining material properties moves in certain 'directions' from it towards the boundaries of the region. I have chosen as coordinates of this representative point:

$$\begin{aligned}
\rho &= 3.3\ \mathrm{g/cm^3}, & A &= 20,\\
\kappa &= 10^{-2}\ \mathrm{cm^2/s}, & B &= 10^5\ \mathrm{s^{-1}\cdot {}^\circ C^{-1}},\\
\alpha &= 2\times10^{-5}\ \mathrm{{}^\circ C^{-1}}, & C &= 0.2\ \mathrm{K^{-1}},\\
H &= 5\times10^{-15}\ \mathrm{cal/cm^3\cdot s}, & T_M &= 1350\ \mathrm{K},\\
C_p &= 0.2\ \mathrm{cal/g\cdot {}^\circ C}, & l_1 &= 0.3\ \mathrm{cm},\\
 & & l_2 &= 10^{-7}\ \mathrm{cm}.
\end{aligned}$$

It would now seem that we have only to find the complete solution of the conservation equations that result from this description of planetary matter (together with Poisson's equation for the gravitational potential) in a simply connected region on which isothermal and stress free boundary conditions have been imposed, and for which the total mass and angular momentum correspond to the various planetary values. Parameter values would then be varied about the representative point in the hope that an interpretation and prediction of planetary observations could be made. Such an obvious approach is unacceptable on several counts. It is grossly inefficient since great labour and expense is devoted to finding details of a solution that can only be compared with observation in a few respects. Although this does not seem to have deterred those who publish pictures of convective flows in planets, what is much more serious is that they convince themselves and others that they are at least solving one particular model with a wealth of detail. Any thoughtful fluid dynamacist knows that his subject is full of solutions that bear no resemblance to the observed situation, and the respectability of fluid dynamical theory has only been preserved by the knowledge or suspicion that the boundary conditions do not specify a unique solution in those cases. Nonuniqueness has profound consequences for the manner in which research on planetary dynamics should be conducted. In the first place, it is self deluding for theoreticians to continue a programme of research that in effect ignores the tremendous diversity of convective flows seen in laboratory systems that come closest to simulating a planetary situation. Secondly, just as I have suggested we rid our theoretical methods of distortions arising from the particular spatial scale of human existence so we need to remind ourselves that our view of planetary behaviour and our judgement of the important unsolved problems of planetary science is continually distorted by the ease with which absurdly detailed and comprehensive spatial maps can be made for an epoch that is utterly trivial in duration compared with the time scales of dynamical behaviour. For planetary physicists to use the indeterminacy inherent in the non-uniqueness of the dynamical problem in an attempt to interprete more than a tiny fraction of what has and could be observed would be equivalent to concerning themselves with the details of the atmospheric weather for a particular day. As suggested elsewhere (TOZER, 1970), the planetary physicist requires, in contrast to the geologist, simple effective methods of evaluating aspects of a convection problem that have little dispersion for the possibly large ensemble of solutions that satisfy the field equations and boundary conditions[5]. Only such invariant features of the solutions can be meaningfully compared with certain aspects of our snapshot view of the planets.

Anticipating the results of many calculations given below, one can make an important simplification in our description of the models. Although it is necessary to use expressions of at least the complexity of eqs. (1) and (2) to give a theoretically complete account of the slow deformation of matter under the variety of conditions one might expect to encounter with planetary models, my experience of calculating horizontally averaged characteristics of the steady state solutions for planetary models with material parameters in the physically plausible vicinity of the representative point indicates that the values of just κ, α, H, C_p, AT_{M} ($\equiv E/k$) and $\frac{1}{4} B\, l_1^2$ ($\approx \nu_0$) effectively determine these aspects of the solutions. These six parameters will henceforth be used to characterise planetary material, and one need only return to the more elaborate specification when demonstrating that the methods of calculation I use give results in substantial agreement with observation if applied to laboratory size systems, and in a sense supercede the criterion of 'solidity' described above.

My first step is to enquire whether the equations and boundary conditions can be satisfied with a velocity field corresponding to simple rotation – the hydrostatic theory of these model planets. In a strict sense this is not a solution for these homogeneous bodies unless the angular momentum is zero (Von Zeipel's theorem). However, the fraction of the heat source density ($= \omega^2/2\pi G\rho$) that is effective in driving a meridional circulation is small ($< 5\times10^{-3}$) for the terrestrial planets, and we may neglect this contribution to the total velocity field unless it is subsequently demonstrated that only a similar or smaller fraction of the heat source density is effective in driving any other mode of convection at finite speeds. It may be mentioned here that although $\omega^2/(2\pi\, G\, \rho)$ is small, it is still very large compared with a measure of the purely dynamical effects of rotation (the effective Taylor number $\nu^{-2}\, \omega^2\, L^4$ for the solutions given below is $\approx 10^{-16}$). It is therefore such indirect effects of rotation[6] that offer the best hope of accounting for any suggestion that the direction of tectonic mass transfer is correlated with planetary rotation.

To the same order of approximation involved in neglecting meridional circulation, one can neglect the ellipsoidal shape of the external surface in the dynamical problem. A very crude calculation of the buoyancy forces that could arise in any body with material parameters near the representative point suffices to show that the viscous stresses are extremely small compared with the hydrostatic portion of the stress at virtually all interior points. We can therefore, to an excellent approximation, impose the isothermal and no-stress conditions on a spherical surface and treat any departures from hydrostatic shape as a height function mapped on this surface or that of the hydrostatic ellipsoid.

It is now a simple matter to write down as a trial solution the steady temperature field corresponding to no motion throughout the sphere. However, just because we now see state of rest or conduction solutions as trial solutions to a class of problems for which the uniqueness of the solutions is unprovable, we have to demonstrate as a necessary condition for physical observability that they are stable to (at least) infinitesimal perturbations. This has been investigated as follows (see also TOZER, 1967): For any spherical region concentric with the external surface and of radius Δ, form the dimensionless quantity ('Effective' Rayleigh

[5] In his concern with short term projections from an elaborate statement of "initial" conditions, the classical geologist plays a role analogous to the weather forecaster in atmospheric physics. This picture is somewhat confused by the unprecedented number of "geophysicists" who now concern themselves with this geological problem.

[6] The latitudinal dependence of temperature on the external surface when the angular momentum vector is not in the orbital plane is another.

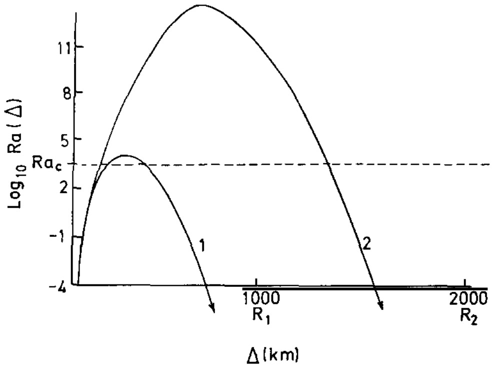

Fig. 1. The Rayleigh number Ra(Δ) as a function of the chosen radius Δ. Curve 1 is for a body ($R \approx$ 1000 km) that only just goes unstable, and curve 2 would correspond, for example, to a larger body ($R \approx$ 1000 km) with the same material parameters.

number)

$$\text{Ra}(\Delta) = \frac{\underline{g}\ \underline{\alpha}\ \underline{\Delta}^3\ (\Delta T)}{\kappa\ \nu_M},$$

where g is a gravitational acceleration characteristic of the chosen sphere (e.g. $\frac{4}{3}\pi\rho G \Delta$) and ν_M the maximum value (found at $r = \Delta$) for the kinematic viscosity within the chosen region. ΔT is the temperature difference between the centre and the surface $r = \Delta$, calculated from the state of rest solution[7]. If

$$\text{Ra}(\Delta) = \frac{g(\Delta)\ \alpha\ \Delta^3\ (\Delta T)}{\kappa\ \nu(\Delta)},$$

where $v(\Delta)$ is the kinematic viscosity value corresponding to the state of rest temperature at $r = \Delta$, is plotted as a function of Δ, we get the typical curve shown in fig. 1. As a test of stability, I use the condition that the maximum value of Ra (Δ) be less than a certain number Ra_c.

Although one may anticipate that an exact statement of the state of rest stability condition for these models would be more complicated than this, one is tempted to use such a criterion because it is known to be a precisely correct form to use for constant viscosity spheres. Certainly it is reasonable to suppose that a sufficient condition for stability could be expressed in this form. There is no pressing need to find an exact value for Ra_c since calculation has shown that the conclusions of this paper are not significantly affected by choices of Ra_c that differ by quite large factors from the exact values computed for constant viscosity spheres (CHANDRASEKHAR, 1961). I have used $\text{Ra}_c = 2.8 \times 10^3$ as a typical value.

This condition of stability for the steady state of rest solutions enable us to divide our models into two classes on the basis of their size. It may be shown that if the material parameters have values plausibly near the representative point, only bodies with external radii less than about 800 km satisfy the condition for stability. One may say of such objects that they lack sufficient internal energy sources to enter a dynamic[8] evolutionary stage of development. Their evolution is just a small contraction induced by the decay of their heat sources. As one might expect, it is the fate of even the largest bodies to become 'dead' in this sense if their heat sources decay, but we shall see that even for models of the size of the Moon (radius $\approx$ 1740 km) this quiescent stage can be delayed much longer than the present age of the solar system. These small objects are interesting in that of all bodies they carry traces of the conditions existing at their birth the longest – both internally and in the external topography. For example, early thermal conditions existing in the solar system are, if at all, best preserved in the larger bodies that have stable conduction solutions.

For the larger bodies in which Ra(Δ) exceeds Ra_c (often by enormous factors) for a finite range of Δ, the temperature field can only be computed along with the velocity field inside the sphere. It is for these bodies that laboratory experiment suggests that many spatial details of the velocity field can become essentially indeterminate through a very high degree of non uniqueness of the solutions to the conservation equations. This is particularly strongly indicated if the solution of the convection problem indicates that mass transfer is effectively confined to a region much broader than it is deep. When we come to consider the introduction of physically plausible effects of pressure on the material parameters characterising the present set of models we shall see that one may reasonably infer that mass transfer is so confined if the external radius of the planetary model significantly exceeds about 3000 km. Fortunately these same laboratory experiments have indicated that some quantities associated with strongly driven

[7] Note that Ra(Δ) is a globally defined aspect of the state of rest solution.

[8] It may be shown that the meridional currents already referred to are so slow for these small bodies that they have negligible effect in disturbing the state of rest temperature profile – their velocities are generally $<10^{-9}$ cm/s.

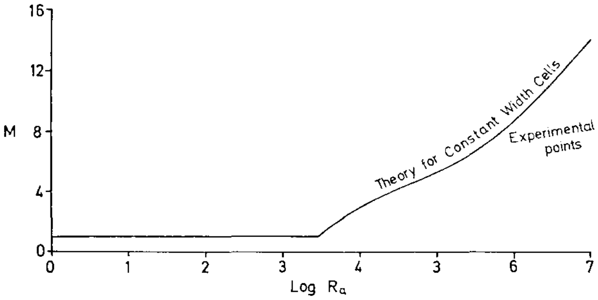

Fig. 2. The theoretically derived function $M = 1$ for Ra < 2800 and $M = 1+\frac{1}{4}$ (Ra$-2800)^{\frac{1}{4}}$ for Ra > 2800, representing the ratio of the state of rest temperature difference across an internally heated plane layer (or sphere) to that which would be associated with motions, M is an "inverse" Nusselt number. For Ra $> 3\times10^5$, the experimental points tend to lie on the low side of the curve. This could be attributed to the increase in horizontal scale of the motions which has not been allowed for in deriving this function and the velocity in the boundary layer becoming progressively less than the average velocity. In any case the magnitude of this discrepancy does not significantly alter the main conclusions of this paper.

(Ra$\gg$Ra$_c$) flows are interesting to the physicist in that they do have values determined within close limits by the boundary conditions. It is on one such quantity, the horizontally averaged temperature difference across a convecting layer of depth 'd' uniformly heated from within, that we shall concentrate. The results of experiments, necessarily performed with plane horizontal layers, and expressed in non-dimensional form by reference to the state of rest solution for the layer, are given as a plot of M(Pr, Ra), where M is the ratio of the temperature dropp across the layer with no motion to the observed (average) drop. The dependence of the function M on the Prandtl number is small when Pr $\gg 1$ and the measurements for Pr ≈ 100 (see fig. 2) are thought to be closely representative of the behaviour at any larger value. To see this and to make the necessary extension to spherical geometry, we only require a rudimentary theory of M. The form of the equations permits us to draw an analogy with viscous boundary layer theory and to introduce a characteristic length δ, the thickness of a thermal boundary layer (Pr$^{\frac{1}{2}}$ gives the ratio of viscous and thermal boundary layer thicknesses). The analogy allows us to suppose that the motion of a parcel of the fluid is adiabatic (apart from its internal heat sources) unless it is within a distance δ of the upper surface. For large Prandtl numbers, the thermal boundary layer is not appreciably sheared and we may think of the quantity M as the ratio of the temperature differences set up across unsheared layers of thickness $\frac{1}{2}d$ and δ by the supply of heat. Hence

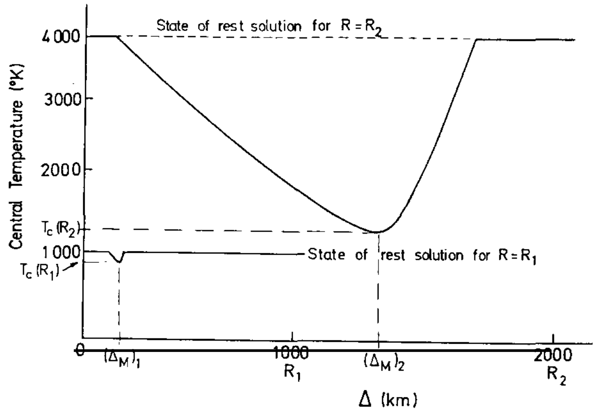

Fig. 3. The central temperature T_c calculated as a function of the chosen radius $\varDelta$ for two bodies $R_1 \approx 1000$ km and $R_2 \approx 2000$ km. The curves are not drawn quantitatively correct.

$$M = \tfrac{1}{2}d/\delta \approx \tfrac{1}{2}d\sqrt{\frac{V}{\kappa x}},$$

where x is a mean horizontal distance traveled by the fluid particles. By quite general arguments (LANDAU, 1959), the velocity is known to vary as $(\kappa/d)(\mathrm{Ra}-\mathrm{Ra}_c)^{\frac{1}{2}}$, and both theory and observation agree on a numerical coefficient of the order 0.25 to compute an average velocity in the layer. Putting $x = d$ and substituting, we have

$$M \approx 1+\tfrac{1}{4}(\mathrm{Ra}-\mathrm{Ra}_c)^{\frac{1}{4}}, \quad \mathrm{Ra} > \mathrm{Ra}_c, \qquad (3)$$

an expression that roughly approximates the data within the observed range of Ra and, as we shall see, the range of principal interest in our present problem.

It should be noted that we have made no assumptions about the form of the velocity field below the thermal boundary layer, except that its horizontal scale be of the same order as the depth. Considering the spherical case, providing δ is much smaller than the radius of the sphere and the variation of fluid properties within the sphere is not such as would significantly restrict the motion to a spherical shell, eq. (3) should also be a good approximation to a function giving the ratio of temperature difference between the centre and surface of a sphere in a state of rest and in a state of motion[9].

[9] In the particular models now under consideration, the decrease of viscosity with depth ensures that all of a sphere with Ra($\varDelta$) $>$ Ra$_c$ is involved in the motion. When we come to consider the plausible effect of pressure, we find that one can sometimes expect the principle motions to occur in a spherical annulus.

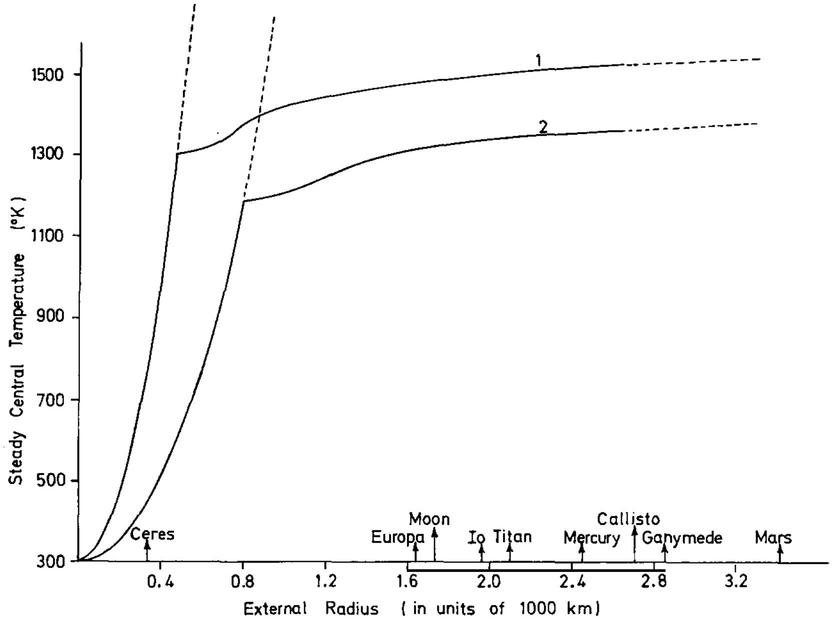

Fig. 4. The (minimised) central temperature T_c as a function of the model external radius for two values of the heat source density: curve 1: $H = 1.6 \times 10^{-14}$ cal/cm$^3 \cdot$ s; curve 2: $H = 5 \times 10^{-15}$ cal/cm$^3 \cdot$ s. Other material parameters are $k = 10^{-2}$ cm^2/s, $\nu_0 = 10^4$ cm^2/s $\cdot$ °C, $E/k = 34800$ °C, $C_p = 10^7$ erg/g $\cdot$ °C, $\alpha = 2 \times 10^{-5}$ °C^{-1}. The steeply rising curves on the left are steady state of rest solutions – unstable where dotted. Points to note are the relative independence of T_c from R when $R \gtrapprox 800$ km and the surprisingly low values of T_c – despite a choice of E/k that is on the high side of the representative point.

In any case, I have made calculations with a plausible range of choices for the numerical coefficient in eq. (3), but again I found that the main conclusions of this paper are not significantly affected.

The function M enables us to calculate for any choice of Δ a central temperature

$$T_c(\Delta) = T(\Delta) + M^{-1}(T(0) - T(\Delta)) = T(\Delta) + M^{-1}\,\Delta T,$$

and if this is plotted, we get the typical curve shown in fig. 3.

When Ra only exceeds $\mathrm{Ra_c}$ for a small range of Δ, the radial extent and general character of the motion is well defined; we note that its Rayleigh number and radial extent may be defined as that which minimises the central temperature $T_c(\Delta)$. For larger bodies in which one finds Ra $(\Delta) > \mathrm{Ra_c}$ for a wide range of Δ, there is a problem to decide the radial extent of the motion. I have treated this problem, at least for these models, by introducing an assumption that the motion is always that which minimises the central temperature[10]. This enables us to define a radius Δ_M for the convective core and an effective Rayleigh number $\mathrm{Ra}(\Delta_M)$ for the motions within it. This is sufficient information to plot a horizontally averaged temperature profile and to calculate such quantities as mean speed, non-hydrostatic stress and kinematic viscosity throughout the core and hence for the whole of the planetary interior.

It should be realised that Δ_M is not meant to define precisely the radial limits of motion, but rather the depth at which the mean temperature profile diverges significantly from the state of rest solution. This distinction is particularly significant when Δ_M differs from the external radius R by only a small fraction of R – as it does for models as large as the Earth. The non-hydrostatic stresses in this outermost shell is in places raised by a factor $\approx R/(R - \Delta_M)$ above the values in the convective zone, and since this can be shown to approach

With appropriate definition of M and Ra, we expect an expression closely approximating eq. (3) to apply in that case also (see TOZER, 1967).

[10] Since the temperature must increase towards the centre, there is always a decrease in the viscosity with depth in these models. In neglecting this variation for $0 < r < \Delta$, I have tended to overestimate by a small factor the temperature increase across the convective core region. However, since the temperature rise calculated with this assumption is small compared with the variations of the estimates of central temperature that follow from different choices of material parameters, this seems justified. However, if the viscosity increases with pressure the procedure can become more complicated (see below).

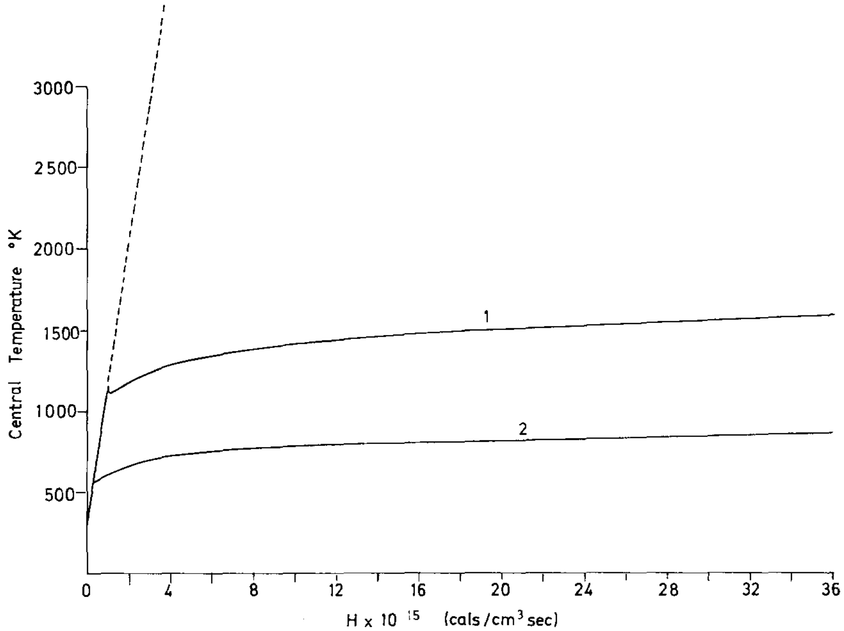

Fig. 5. The central temperature T_c as a function of the heat source density H for a body of external radius 1740 km. The two curves bifurcating from the state of rest solution on the left are for two materials that bracket the plausible range of creep behaviour: curve 1: $\nu_0 = 10^4$ cm^2/s · °C, $E/k = 34000$ °C; curve 2: $\nu_0 = 10^2$ cm^2/s · °C, $E/k = 21000$ °C. Other parameter values are as in the caption to fig. 4. One should note how small are the values of H for which a stable state of rest solution can be found, and the independence of T_c from H, the "boiling point" phenomenon referred to in the text. The temperature difference of curves 1 and 2 almost exactly compensates for the different viscosity–temperature relationships.

the values that one might plausibly give for the shear modulus μ of this layer, we do not expect its deformation to be adequately described by the present viscous models. It is unfortunate that a precise dynamics of the region most accessible to observation is still beyond us, although certain general ideas about its behaviour can be suggested (see also below).

It should be added that although various arguments can be advanced in support of the assumption about minimising $T_c(\Delta)$ – not the least being the inconsistencies that arise unless it is approximately correct – it cannot be proved from the basic equations of the problem. Perhaps the only way to judge such a tremendous simplification of a very difficult problem is by the conclusions that follow from its use. Certainly these are not critically dependent on it being precisely true.

4. Discussion

Some results of the calculations of the steady slow deformation problem for the homogeneous spherical models described above are shown in figs. 4–7. These should be sufficient to give the reader some insight into the behaviour of these highly non-linear problems, and to enable him to make a rapid assessment of the solution for his own choice of values for the material parameters. By containing the first real attempt to assess the influence of a temperature-sensitive rheology on the level of absolute temperature, such models are the first steps towards a radically new approach to planetary thermal problems.

It is immediately evident from fig. 4 that once a model has been chosen large enough for its central viscosity to be incapable of stabilising a state of rest solution, the steady central temperature T_c becomes comparatively independent of the model radius when R $\gtrapprox$ 800 km – and hence diverges rapidly from the values based on state of rest or conduction theory arguments. In these larger bodies ($R \gtrapprox$ 800 km) that have finite motions, there is also little dependence of the absolute temperatures in the convective core on the exact value one chooses from the plausible ranges of some of the material parameters.

An example of this is shown in fig. 5 by a plot of the central temperature against the heat source density H

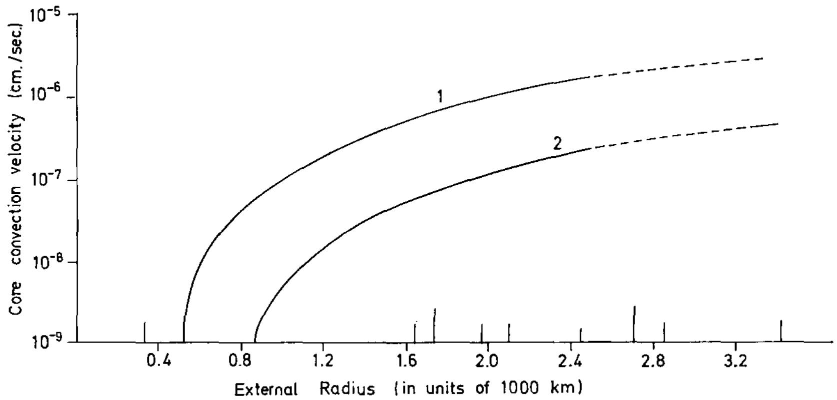

Fig. 6. Mean convection velocities Δ_M/R in the convective cores as a function of the external radius R. The trend to higher velocities in larger bodies would be halted, if not reversed, by the evolution of the convective core into a convective shell by pressure effects. This has not been considered here. There is a general order of magnitude agreement at planetary sizes with the velocities associated with the continental drift hypothesis for the Earth.

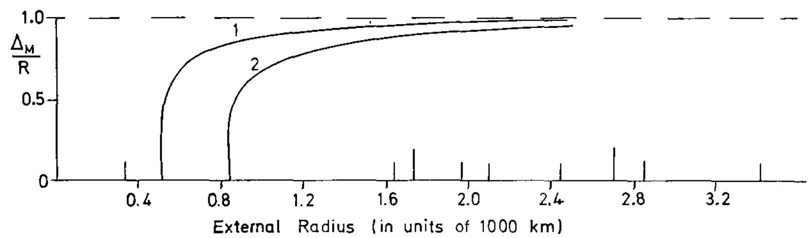

Fig. 7. The radius of the convective cores (see text) expressed as a fraction of the external radius, for bodies of different external radius. The two curves refer to the same two sets of material parameters used in computing fig. 4. Notice how rapidly a convective core increases in size once a state of rest solution becomes unstable. This reflects the difficulty of starting convection in a small central region of a self gravitating object. This ability of the scale of motions to grow in the direction of gravitational acceleration accounts for the central temperature being so insensitive to heating rates – unlike a laboratory experiment where the vertical scale is fixed by the bounding surfaces, i.e., chemical inhomogeneity. It will now be seen how restricted is the region of planetary interiors in which a state of rest solution has any validity.

for a model with approximately the lunar radius and for two materials with viscosity parameters ν_0 and E/k that one might take as bracketing their plausible values – particularly of E/k. Beyond a value of H at which the state of rest solution becomes unstable, the central temperature changes relatively slowly – there is even a small decrease at first that can be attributed to the difficulty of starting a small scale convective motion at the centre of a self-gravitating object. Qualitatively, one may describe this behaviour as a quasi boiling point phenomenon – after an initial rise of temperature, additional heating mostly makes the convective core larger and its motions faster.

In rather more precise terms, one may ask what are the material parameters which, if their values are varied within plausible limits, significantly alter this 'plateau' or 'saturation' phenomenon of the temperature distribution. A clue to this is contained in fig. 8, which shows, for two materials with heat source densities differing by a factor $\approx$ 3, but otherwise identical material parameters, the kinematic viscosity at the radius Δ_M of the convective core as a function of the external radius of the model[11]. This clearly shows that any plausible heat source density does not reduce the convective core viscosities of these bodies much below $\approx 10^{18}$ stokes.

Other calculations show this particular statement to be correct when the other material parameters are varied throughout their plausible ranges. For example, the two materials chosen to plot fig. 5 have viscosities at, say, 750 K that differ by a factor of more than 10^9, but the viscosity ratio within convective eores composed of these materials is, for plausible values of

[11] For the smallest bodies with no convective cores it is the kinematic viscosity at the centre of the body.

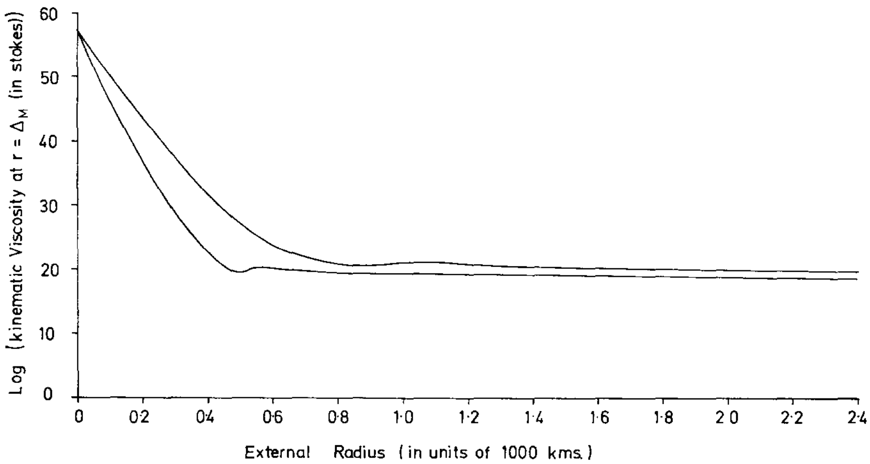

Fig. 8. The kinematic viscosity either at the centre of a model planet or at the surface of its convective core $r = \Delta_M$ as a function of external radius. The two curves correspond to the two heat source densities used in fig. 4, and the other material parameters are given similar values. This is an attempt to show how the kinematic viscosity controls the thermal situation once a value $\approx 10^{22}$ stokes has been attained inside a planetary object due to increasing size or heat source density. As mentioned in the text, this rule applies if other material parameters are plausibly varied.

H, between 10 and 100. This behaviour puts a completely new slant on the problem of temperature determination. In a crude way, one can say that for any plausible material the average temperature rises with depth according to a conduction or state of rest solution until either the centre of the body is reached or the kinematic viscosity has fallen to values $\approx 10^{20}$ stokes – whichever is reached first. With any plausible choice of v_0, such viscosities are reached somewhere between 0.5 T_M and 0.8 T_M. It may be noted that the effect of changing the thermal conductivity or the temperature of the external surface is principally to alter the depth of the boundary of the convective core[12]. The latter effect may be taken as an indication that differences of temperature between equator and pole on the external surface distort the spherical core rather than impress horizontal temperature differences on its boundary $r = \Delta_M$.

The reader who may be puzzled by the quasi 'boiling point' being determined by a particular value of the viscosity should realise that it is only an approximate way of expressing a complicated non-linear behaviour. For example, if one were to consider objects of decreasing size, one finds that the heat source density required to reach the temperature 'plateau' increases slightly faster than R^{-2} and the defining viscosity to continuously decrease. For bodies less than about 10 km radius, the plateau is defined by a value of viscosity in the range in which it is changing enormously fast (due to the factor tanh $C(T_M - T)$, i.e., the plateau temperature is $\approx T_M$. For laboratory sized objects, no significant motion is ever thermally induced below T_M – even in the 'externally' applied gravitational field of the Earth. This combined with the fact that motions are readily induced if $T > T_M$ (due to l_2 being so much smaller than l_1) enables us to assess plausible values for T_M by the 'simple empirical procedures' mentioned in the introduction.

[12] Since the characteristics of the core motion are determined by the effective Rayleigh number, a product of physical parameters including viscosity, one can see that it is a misdirection of effort to try and further reduce the relatively small plausible ranges of k, α, C_p and H.

5. Time-dependent behaviour of the homogeneous models

So far, we have been concerned with steady solutions to the slow deformation problem, and one may ask if a good case can be made for thinking that the present thermal state of the terrestrial planets approximates such solutions. We can discuss this by estimating the time various postulated temperature distributions take to relax towards the steady solutions.

Let us first consider a distribution lying well below the steady state distribution – for example an isothermal state with $T = 300$ K. We may now use the fact that for a plausible choice of material parameters the viscosity rises very rapidly below the steady state distribution – at 300 K it is $> 10^{50}$ stokes – and we are therefore quite justified in using conduction theory to study the relaxation processes to within temperatures of the order 100 K of the steady state. The plausible heat source densities and specific heats set characteristic heating rates ≈ 1 °C every few million years, and at that rate the relaxation process would take $\approx 10^9$ y. If such an isothermal state existed 4.5×10^9 y ago and we make the conventional identification of the heat source density with the activity of certain radioactive isotopes, then steady state conditions would then have been reached in times $\approx 2 \times 10^8$ y. Such a calculation gives us confidence in believing the present thermal state would not now reflect a 'cold' origin of these bodies.

Let us now consider the relaxation from a distribution above the steady state – e.g. an isothermal distribution with $T = T_M$. In this case, the initial viscosities are more than 10^6 times less [13] than those associated with the steady state, and one must expect a relatively vigorous motion to be set up ($V > 10^{-4}$ cm/s). I have made an estimate of the relaxation time by considering the cooling as a sequence of quasistatic states. Firstly, one computes the steady state temperature distribution as a function of increasing heat source density until one reaches a temperature distribution that closely approximates the initial temperature distribution (in this case T_M) throughout the interior[14]. A cooling rate at a particular temperature is then defined as the difference between the heat source density required to maintain it and that which supports the final steady state distribution, divided by the specific heat per unit volume. Such calculations indicate that the cooling time from $T = T_M$ to a distribution everywhere within about 100 °C of the steady state varies approximately as the square of the radius of the model up to a radius of the order 600 km at which it is more than 10^9 y and thereafter declines to a value $\approx 2 \times 10^8$ y for bodies having an external radius greater than about 1500 km. Since these times are shorter than the half lives of the principle heat producing elements – as well as the expected age of the terrestrial planets – I conclude that steady state calculations are a good approximation to the present thermal state of these bodies.

[13] For material near the external surface, the factor is very much greater than this.

[14] The convective core, which is always nearly isothermal inside the relatively thin thermal boundary layer, grows outward as H is increased and when its temperature has reached T_M, it is virtually the whole body. The motions are so rapid that the quasistatic treatment seems reasonable. It will be seen that the cooling from some initial distributions, e.g. those with $dT/dr > 0$, are more awkward to calculate.

6. Extensions of the homogeneous models

With the knowledge we now have about the behaviour of models with temperature dependent viscosities, we can discuss two plausible additions to their definition which in many cases amount to little more than small refinements to the above methods of calculating the present thermal state.

The first I wish to discuss is the rather diverse ways in which the radial pressure gradient of the models could modify the steady solutions we have calculated for the basic homogeneous models. This can happen if:

(1) the temperature changes purely by a pressure change – obviously only involved in those solutions that have some radial motion;

(2) the six material parameters that essentially determine the averaged properties of the steady state solutions (see above) are regarded as varying with pressure.

The first of these effects is in fact already present in the basic models, since

$$\left(\frac{\partial T}{\partial P}\right)_S = \frac{\alpha T}{\rho C_P} \neq 0 . \tag{4}$$

Given the assumptions about the constancy of material parameters in those models, one may integrate eq. (4) to find the temperature rise δT if material were to travel all the way from the centre to the surface – an upper limit to the effect. We have, with $T \approx 1300$ K,

$$\delta T = \tfrac{2}{3}\, \pi\, G\, C_P^{-1}\, \alpha\, T \rho\, R^2 \approx 1.2 \times 10^{-15}\, R^2 . \tag{5}$$

Even this upper limit for the temperature rise does not become comparable with the range of possible estimates of central temperature (≈ 400 K – mainly due to the range of plausible values for the parameters ν_0 and E/k) until $R \approx 5700$ km. In fact more detailed calculations have been made which use the theoretical result that the Rayleigh number should be defined in terms of the superadiabatic temperature difference across a layer. If the minimising procedure to find the central temperature is then performed, it may be shown that the systematic error introduced by ignoring compressional heating is much smaller than the upper limit given by eq. (5).

Of course, this effect of compression has to be recalculated if what I have loosely called the second effect of pressure introduces a systematic variation of the relevant material parameters with depth. From what we know from laboratory experiment to set plausible ranges for the pressure coefficients of the parameters κ, α, H, C_p, (E/k) and ν_0,[15] one may say that for bodies less than about 3000 km in external radius the solutions may be expected to depart insignificantly from that for some constant material parameter model taken from the plausible vicinity of the representative point. For bodies larger than this, the central pressure has risen to values at which it is plausible to assume that the density has been increased by more than 10%. Not only does this produce change from any constant parameter solution but it is also important to enquire whether such density increments are associated with significant changes in the values of the other material parameters that define the slow deformation problem.

Let us first assume that the density and pressure distributions for the Earth's mantle define the compressibility of planetary material at pressures up to $\approx$ 1.5 Mbar and that there are no changes in the other material parameters (α, κ, ν_0, E/k, C_p, H). Calculations by PELTIER (1971) of the effect of finite compressibility on the stability of the state of rest of plane layers suggest that the total increment of density during compression to the Mbar range is insufficient to significantly change results based on an assumption of zero compressibility, and we may reasonably suppose that the effect of compressibility *alone* is unimportant for bodies of the size of the terrestrial planets.

What appears to be most important are the changes in the value of viscosity that are plausibly associated with the density increases. Let us just consider the variation of E/T with depth and express the local rate of change of temperature with pressure as a multiple θ of $(\partial T/\partial P)_S = \alpha T/(\rho C_p)$. Then we have:

$$\frac{\mathrm{d}}{\mathrm{d}z}\frac{E}{T} = \frac{1}{T}\frac{\mathrm{d}E}{\mathrm{d}z} - \frac{E}{T^2}\frac{\mathrm{d}T}{\mathrm{d}z}$$

$$= \frac{E}{T}\frac{\mathrm{d}P}{\mathrm{d}z}\left(\beta\frac{\mathrm{d}\log E}{\mathrm{d}\log V} - \frac{\theta}{\rho}\frac{\alpha}{C_P}\right) \qquad (6)$$

$$= \frac{E}{T}\beta\frac{\mathrm{d}P}{\mathrm{d}z}\left(\frac{\mathrm{d}\log E}{\mathrm{d}\log V} - \theta\,\gamma\right).$$

Since this quantity is only involved (through the viscosity) in calculations of the mean temperature profile for the convective core, changes in the value of E/T above this region have, in effect, been taken care of by a suitably broad range of choices for E. What is of interest is whether the values of $\mathrm{d}(E/T)/\mathrm{d}z$ for $r < \Delta_M$ significantly alter the solution for that region. For the solutions of the constant parameter model θ are only marginally greater than unity – particularly in that part of the convective core below the thermal boundary layer – and since values in the range 1.5–2.0 can be plausibly assigned to both d log E/d log V and γ, it is reasonable to believe that the effects of compressibility on the kinematic viscosity are effectively cancelled by the rise of temperature with depth. Such perfection of cancellation is destroyed if relatively rapid increases in density associated with phase transformation occur (cf. the interpretations of the density profile in the Earth's upper mantle).

Although experimental data indicate that neither γ nor d log E/ d log V would be systematically affected by phase transformation, there are important changes in β and θ. As explained in TOZER (1959), during the occurrence of a first order phase change the volume changes and latent heats of the transition augment the normal expansion coefficient and specific heat. For the particular phase transitions thought to occur in the Earth's mantle material, α/C_p and hence the local adiabatic gradient is temporarily increased by a factor of the order ten. In the same range of depths, the compressibility β in eq. (6) is also increased by a factor $\approx$ 10.

One can assess the importance of these abnormal values in modifying the constant parameter solutions by looking for possible inconsistencies of an assumption that the pressure–temperature relationship remains unchanged. The value of θ drops to $\approx$ 0.1, so that the phase transition region is stably stratified and can only be disturbed by a sufficiently vigorous 'penetrative' convection. The expression in parentheses in eq. (6) becomes positive, and with the effect of a large β, one calculates for a transition region similar to that which

[15] One has to be a little careful about ν_0. The above remarks are based on an estimate of the changes in l_1, associated with lattice compressibility. However, l_1, which might in some circumstances be called the "crystal size", is not fixed by the local thermodynamic variables but the past history of the material. Some authors have asserted that in a planetary interior l_i might be enormously larger than my choice based on the observation of surface material. This is not strictly a pressure effect and I ignore it. It does raise again the question of how much one should ignore laboratory observations.

is thought to exist in the Earth's mantle that E/T would increase by $\approx 40\%$. This would imply an increase of kinematic viscosity by a factor $10^5 - 10^6$, i.e., to a kinematic viscosity $\approx 10^{25} - 10^{26}$ stokes.

An inconsistency now exists since such a value of kinematic viscosity is sufficient to stabilise a steady state of rest solution for the spherical region that could exist below the phase transition region – even if it were some thousands of kilometres in radius. Furthermore, the combined effects of the viscosity increase and stable stratification prevent significant 'penetrative' convection into this region from above. I conclude that if one considers a set of models of increasing external radius with material parameters near the representative point (defined above) but with the addition of a plausible phase transition, we find that the idea of a convective core is only applicable for bodies in the range 800 km $\lessapprox$ R $\lessapprox$ 3000 km. For larger bodies, the motions become effectively confined to a spherical shell whose outer boundary grows only very slightly faster than the external radius,[16] but whose internal boundary (fixed by the depth of phase transition) grows relatively rapidly since the mean pressure gradient in a model is nearly proportional to its external radius.

As one might have suspected from knowledge of the solutions for smaller bodies with convective cores, the absolute temperature of the convecting shells is essentially fixed by the viscosity attaining a value $\approx 10^{20}$ stokes. (The particular case of the lower boundary being 600 km deep was considered in Tozer (1967) as an Earth model.) In fact, it is reasonable to treat the interior region of the planet as a separate problem whose 'external' temperature is fixed by the viscosity–temperature relationship of the material outside it. Although the existence of chemical inhomogeneity in the larger terrestrial planets may make it an academic point, it seems that for the above models with external radii greater than about 5000 km there could be a central convective core as well as a thin convective shell no more than about 750 km thick near the external surface.

The only other extension of the constant parameter models that I wish to discuss here is the spatial differentiation and decay of the heat source density. This is not the place to discuss in detail how differentiation can occur. However for the sake of what follows one needs to say that the only plausible location for its initiation and development are a few highly localised regions of very much lower viscosity than average that can be predicted to occur as a result of viscous dissipation of the motions in models that exceed about 1000 km in external radius. This exclusive association of heat source differentiation with passage of the primitive material through a low viscosity state rests on the idea that the heat sources are radiogenic nucleii partitioned between phases of different density; such low viscosity zones are the only places in which the relative velocities of the different density phases can plausibly be expected to exceed the convection velocities – a necessary condition for differentiation to develop. In this paper, I shall neglect all aspects of the differentiation process other than the geochemically plausible tendency of the heat sources to move upwards with the least dense phases. I shall assume that the differentiation process is slow in the sense that at all times the convective motions themselves preserve an effective uniformity of the heat source distribution within a zone of convection.

The first point to notice is that, in comparison with the state of rest solutions, the temperature distribution is relatively insensitive to the heat source density (fig. 5). Therefore, the upward motion of quite large fractions of the initial heat source distribution ($\approx 5/6$ for a body of the size of the Moon and even more for larger bodies) can occur before the central temperatures is significantly affected, and a state of rest becomes a stable solution of the slow deformation problem. However, the complete cessation of motion throughout the interior would be only the last point of a steady downward migration of the outer boundary of the convective zone. During that migration, the external surface of the body becomes progressively less susceptible to disruption by the stresses associated with motion in the convection zone. Furthermore, the source regions of the differentiation process necessarily move downward, i.e. to a level of higher hydrostatic stress, and this could lead to a systematic change with time of the mineralogy and composition of the differentiating material[17].

From the amount of energy that is available for viscous dissipation ($g\,\alpha\,L\,(H - H_c)/C_p$ per unit volume)

[16] When $R - \Delta_M$ is small compared with R, as it is for models with $R \gtrapprox 1500$ km, we have $R - \Delta_M$ approximately proportional to R^{-1}.

[17] Due to the effects of differentiation on the viscosity–temperature relationship, the problem is more complicated than indicated here. It will be discussed at greater length in a separate paper.

it may be calculated that the rate at which material could pass through a low viscosity state in which separation might plausibly occur ($\nu \approx 10^6$ stokes?) is not greater than a few km^3/y, so that the fractional rate of change of the mean heat source density of the convection zones would be no more than 10^{-9}–10^{-10} y^{-1}, comparable with the rate of decay of ^{40}K, but quite sufficient to produce important changes in the position of convection zones over a few milliard years.

7. Conclusions

My concern in this paper has been to assess some of the consequences of assuming that the material forming the terrestrial planets has a plausible degree of creep resistance. What is perhaps most remarkable about this single assumption is that the thermal problem of planets, indeed any system, is immediately seen to be embedded in and merely one facet of a much larger problem of dynamics. The mathematical structure of this theory is fundamentally different as well as of a different order of complexity from the classical theory of heat conduction that has dominated thinking on planetary thermal problems. Difficult and subtle questions about the uniqueness and stability of a solution must be answered to be confident that ones results are not only quantitatively correct but physically important. As has happened in atmospheric dynamics – another branch of geophysics with the same difficulties – the degree of non-uniqueness strongly influences one's selection of observations worthy of a quantitative physical interpretation and which observations are, like today's weather, the province of the data collector and forecaster. It should also curb the impulse to produce detailed numerical solutions of these problems.

All conclusions in this paper are based on the representation of the creep behaviour of planetary material by a Newtonian viscosity, exponentially varying with temperature. To those who feel that such an assumption is a priori illegimate I would say that the scientific method only judges hypotheses by the results that they produce. I have indicated that the Newtonian theory is best seen as the theory of slow deformation and is complementary to the Hookean postulate covering fast deformations of materials with fading memory. The latter has been used with great success in some branches of seismology without protestations that it is not the 'true' behaviour of 'real' materials. I would only remark of the observations and theory of creep behaviour that strain rate versus stress curves are convex towards the stress axis and that the estimates of plausible Newtonian viscosities in this paper are based on estimates of the slope of the tangents to such curves at the origin. The inclusion of non-Newtonian behaviour will tend to increase rather than diminish the already big discrepancies that exist between the conclusions of this paper and the results of heat conduction theory appropriate for an infinite creep resistance.

An important conclusion of this paper is the manner in which the sensitivity of the steady temperature field to different choices of the material parameters depends strongly on the external radius of the model body. Having regard to the finiteness of the range of plausible choices, one may say that if the external radius is less than about 800 km the heat source density and thermal conductivity are the only material parameters defining the internal temperature. For larger external radii, this exclusive dependence on two parameters becomes confined to a shell of rapidly diminishing thickness that forms the outermost parts of the body (fig. 7). The temperature in the central region is determined within close limits by the choice of a viscosity–temperature relationship since its steady kinematic viscosity is always in the vicinity of 10^{20} stokes. Within this region of finite motions, which changes to an annulus of only some hundreds of kilometres thick on the outside of the model planet as the choice of its external radius is increased from about 3000 km to $\approx$ 6000 km, one calculates that the mean speed of motion is 10^{-6}–10^{-7} cm/s and the mean nonhydrostatic stresses are 1–10 bar. It should be noted that these particular figures do not vary significantly for the numerically plausible ranges of the material parameters.

A study of time-dependent behaviour suggests that the evolution of a model from any one of an extremely wide range of initial conditions would bring it close to the steady solutions in times no greater than a few 10^8 y. Such a rapid response compared with the time constants of radioactive decay and that expected for spatial differentiation suggests that steady state arguments describe, as for stars, a 'main sequence' of evolution. Only bodies with masses greater than about 10^{25} g can enter this particular main sequence if they have an approximately meteoritic composition. If they are of the size of the terrestrial planets, they spend rather more than 10^{10}–10^{11} y passing through this sequence before becoming 'dead' objects[18]. Since the capacity to extensively differentiate would seem to be wholly dependent on the body entering this 'main sequence', the creation as well as the retention of an environment suitable for the development of life is probably confined to bodies of at least a few 1000 km radius.

There is no need for specific comment about the viscosities, velocities and non-hydrostatic stresses that have been derived for models of planets other than the Earth, except to say that one's confidence in these predictions stem directly from their insensitivity to the choice of numerical values for material parameters. At least, we can now say that a single class of plausible Earth models adequately accounts for diverse geophysical observations.

Regarding the predicted horizontally averaged temperature fields of models representing different members of the solar system, we can say that the first real test of these ideas came from lunar research where calculations had indicated that, even with very optimistic estimates of resistance to creep, the internal temperatures should be rather less than ≈ 1000 °C. Studies of lunar electromagnetic induction have indicated an internal electrical conductivity distribution that is only reconcilable with such low temperatures. The case of the Earth is interesting in that at a time when the motion of outer portions of the Earth is widely accepted, popular ideas on the thermal state of its interior date from an epoch when no motion was dogma.

If we dismiss the conduction theory arguments on the grounds of their inconsistency with current geophysical ideas on dynamics, we are left with a few indirect observations that do not significantly conflict with my prediction that horizontally averaged temperatures down to depths of several 100 km do not exceed 1000 °C – they could even be several 100 °C less. For example, interpretations of the varying magnetic fields at the Earth's surface that are dependent on an internal electrical conductivity distribution have been unable to place more than an upper limit on the value of horizontally averaged electrical conductivity at these depths, and this in turn places a plausible upper limit of ≈ 1200 °C on the temperature (TOZER, 1970).

Lava production has sometimes been invoked as evidence that upper mantle temperatures are just below the solidus – a concept (like 'partial melting') that is a left over from the days when it was thought possible to classify matter in a few mutually exclusive rheological categories. The weakness of this argument is particularly apparent when it is remembered that the new theory predicts that low viscosity material at much higher (several 100 °C) temperatures than the average would be produced by viscous dissipation and in amounts similar to that estimated as total lava production by vulcanologists. The evidence from seismology that partial melting is occurring in the upper mantle I dismiss on the grounds that with a modern view of material rheology one cannot place any limits on the temperature with such a concept.

There are some implicit features of these solutions that should be briefly mentioned. The first two reflect on the way the current craze for geokinematics is confusing important issues, by using everyday language to express the behaviour of a very unfamiliar domain of dynamical behaviour. It is readily calculated that the effective Reynolds number of the solutions is extremely small ($\approx 10^{-18}$), and it follows that, unlike the thermal boundary layers, a purely formal calculation of viscous boundary layer thicknesses gives lengths much greater than the overall dimensions of the system. A little reflection should convince the reader that this result, expressed in everyday language, would be that every part of the system is tightly coupled to the rest. The notion that the deeper parts of a planetary model are 'decoupled' from the surface material is a dynamical nonsense. Similarly, the concept of a 'collision', i.e. coupling limited in time and space, needs clarification.

I have already mentioned that a test of the consistency of a viscous model for the same region to which elastic parameters are also assigned is that the strain be very small in a time τ ($= \eta/\mu$). For all planetary models, there is a surface region – rather thinner than $R - \Delta_M$ – in which this condition cannot be satisfied. In a lecture I gave in Toronto in 1964, I pointed out that the division of the Earth into regions that did or did not pass this consistency test was far more dynamically significant

[18] In a star, a precise thermal equilibrium is maintained in the presence of a temperature-sensitive heat production by the capacity of the object to change heat production by a change in its size. For a planet, thermal equilibrium in the presence of fixed heat sources is maintained by a highly temperature-sensitive process of heat transmission to the outside.

than the Mohorovičić discontinuity then favoured. The plausible inference that in regions in which consistency does not exist the deformation may be crudely described as domains of rigid body motion has been confused by the concept of 'rigid' plates, whose most notable characteristic seems to be a tendency to change size and bend. Of course, words are not in themselves important, but this description of an instantaneous kinematic situation by a rheological concept has persistently given the impression that the surface motions are more highly constrained than is in fact the case.

The other point I wish to consider is the existence at the present time of a 'liquid' core in the Earth and possibly other bodies. Although the normal interpretation of such regions is that the shear modulus is zero, I shall take the position that the shear modulus is not changed by a large factor; but 'liquidity' means that $\tau = \eta/\mu$ is much less than the period of seismic disturbances. Such an interpretation implies the maintenance for milliards of years of viscosities very much less than those characterising steady solutions of the slow deformation problem. It may be shown that the only possible reasonable way would be for the region outside the 'liquid' core to have an activation energy E (or a T_M) for the viscosity more than 30% greater than that for the core. If we are to believe the estimates of T_M for the lower mantle, we cannot satisfy this inequality with the T_M suggested for a pure iron core. The inequality can be satisfied if some T_M reducing element, such as sulphur, is mixed with the iron.

Explanation of symbols

A. B, C	material parameters appearing in relations expressing the dependence of kinematic viscosity on temperature
C_p	specific heat at constant pressure
E	thermal activation energy for the creep process
G	gravitational constant
g	acceleration of gravity
H	heat source density
H_c	heat source density at which motion is initiated
k	Boltzmann's constant
L	characteristic length
l_1, l_2	parameters expressing the range of structural ordering in planetary material for $T \ll T_M$ and $T \gg T_M$
M	ratio of the temperature differences across a plane layer (or sphere) for the state of rest solution and for that actually observed (or expected to be observed). One might call it an inverse Nusselt number
P	pressure
Pr	Prandtl number ($= \nu/\kappa$)
R	external radius
r	radius variable
Ra(Δ)	Rayleigh number based on a state of rest solution for a sphere radius Δ
Ra_c	critical value of Rayleigh number used to mark the onset of instability of a state of rest
S	specific entropy
T	temperature
T_c	temperature at the centre of a planet
T_M	parameter appearing in viscosity relationship to thermodynamic conditions
ΔT	temperature difference between the centre and $r = \Delta$ for a state of rest solution
V	volume
v	velocity of convective motions
α	coefficient of volume expansion
β	compressibility
γ	Grüneisen's parameter ($= \alpha/(\rho\ \beta\ C_p)$)
Δ	radius variable used in computing Rayleigh numbers
Δ_M	value of radius variable Δ for which one computes a minimum central temperature
δ	thermal boundary layer thickness
η	coefficient of shear viscosity
κ	thermal diffusivity
ν	kinematic viscosity ($= \eta/\rho$)
μ	shear modulus
ρ	mass density
τ	stress relaxation time
ω	planetary angular velocity

References

Chandrasekhar, S. (1961), *Hydrodynamic and hydromagnetic stability* (Oxford Univ. Press, London).
Landau, L. D. (1959), *Fluid mechanics* (Pergamon, London) 103.
Peltier, W. R. (1971), Ph.D. Thesis, Univ. of Toronto, to appear.
Tozer, D. C. (1959). In: *Phys. Chem. Earth* **3**, (Pergamon, London) 414.
Tozer, D. C. (1967). In: T. F. Gaskell, ed., *The Earth's mantle* (Academic Press, New York) 325.
Tozer, D. C. (1970), J. Geomagn. Geoelec. **22**, 35.
Tozer, D. C. (1972). In: *The Moon* **5**, (Reidel, Dordrecht) 90.
Truesdell, C. and W. Noll (1965). In: *Handbuch der Physik* III/3 (Springer, Berlin).