ENERGY SOURCES FOR THE GEODYNAMO

The magnetic field of the Earth is maintained against ohmic losses by dynamo action in the fluid core which takes its energy from several sources of different natures and amplitudes, all being consequences of the thermal evolution of the core. Moreover, each source has a different efficiency in maintaining ohmic dissipation compared to the others. The study of these problems is then of major importance for understanding dynamo theory as well as the thermal evolution of the Earth.

Evolving reference state

The core of the Earth is composed of iron, nickel and some lighter elements, with concentrations that are still matter of lively debates (see Core Composition, and Poirier (1994)). The light elements play a very important dynamical role in maintaining the geodynamo since their rejection upon inner core crystallisation drives compositional convection in the core (see Convection, chemical). The core is therefore usually modelled as a binary alloy of Fe and some light element X. Including a more realistic chemistry in thermal evolution models on the core would add complexity (although no real difficulty) in the derivations without really improving the understanding, unless not yet documented coupled effects occur.

The Earth's core is different from a well controlled convection experiment in a laboratory, in the fact that it is continually evolving on geological time scales and that the energy for the motion comes from that evolution. Fortunately, the time scales relevant for this evolution and that for the dynamics are very different and can be separated. To get expressions for the different energy sources in the balance, we need to know in what state is the core when averaged on a time scale long compared to the one relevant to the dynamics but short compared to the one associated with the thermal evolution. The short time dynamics is responsible for maintaining the core close to that reference state whereas the evolution of the reference state provides the energy sources needed by the dynamics.

Convection in the liquid core is assumed to be very efficient so that, outside of tiny boundary layers (see Boundary layers in the core), all extensive quantities responsible for the movement (entropy s and mass fraction of light elements ξ) are uniform: $\nabla\xi = \mathbf{0}$ and $\nabla s = \mathbf{0}$. In addition, the momentum equation is assumed to average to the hydrostatic balance: $\nabla p = \rho\mathbf{g}$, with ρ the density and $\mathbf{g}$ the acceleration due to gravity. It is often useful to know what the temperature is in the reference state (see Temperature of the Earth's core) and isentropy and iso–chemistry implies it to be adiabatic (see Adiabatic gradient in the core), $\nabla T = \alpha\mathbf{g}T/C_p$, with α the coefficient of thermal expansion and C_p the heat capacity at constant pressure.

The set of partial differential equations defining the reference state must be supplemented by an equation of state relating the density to the pressure, for example at constant entropy, and by boundary conditions. In terms of temperature, the equilibrium between solid and liquid at the inner core boundary (ICB) provides the required condition, the liquidus of the core alloy being given (see Melting temperature of iron in the core, theory and Melting temperature of iron in the core, experimental) as a function of radius in the core. Depending on the choice of equation of state one can get different expressions for the different profiles in the reference state, usually in the form of some series expansion of powers of the radius.

It should be noted that the actual averaged state cannot be exactly hydrostatic, owing to its being compressible and compressing with time, but the corrections to this balance are negligible for the energetics of the core (Braginsky and Roberts, 1995). The deviations from the isentropic, well mixed state can also be estimated from the amplitude of fluid velocity at the top of the core which also allow an estimate of the typical scale for the size of boundary layers. These are found to be a negligible contribution to averaged quantities (e.g. Braginsky and Roberts, 1995), although they are very important for the dynamics of the core.

Energy equation

The equation for total (internal, kinetic, magnetic and gravitational) energy conservation can be integrated over the volume of the core and it states that the total heat loss of the core, the heat flow across the core mantle boundary (CMB), is balanced by the sum of four terms, the secular cooling Q_C associated with the heat capacity of the core, the latent heat Q_L associated with the gradual freezing of the inner core, the gravitational energy E_G associated with the rearrangement in the outer core of the light elements that are released at the ICB by the freezing of the inner core, and possibly the radioactive heating Q_R,

$$Q_{CMB} = Q_C + Q_L + E_G + Q_R. \qquad (1)$$

The first three terms are related to the growth of the inner core and can be written as a function of the radius of the inner core c times its time derivative. The exact expression of these three functions depends on the choice of equation of state and parameterisation and the expressions of Labrosse (2003) can be written to first order as:

$$Q_C = \frac{8\pi}{3} b^3 \rho_0 C_P T_L(c) \left(1 - \frac{2}{3\gamma}\right) \frac{c}{L_T^2} \frac{dc}{dt}, \tag{2}$$

$$Q_L = 4\pi c^2 \rho(c) T_L(c) \Delta S \frac{dc}{dt}, \tag{3}$$

$$E_G = \frac{8\pi^2}{3} G \Delta\rho \rho_0 c^2 b^2 \left(\frac{3}{5} - \frac{c^2}{b^2}\right) \frac{dc}{dt}, \tag{4}$$

with b and c the radii of the outer and inner core respectively, ρ_0 the central density, C_P the heat capacity, T_L the liquidus temperature, γ the Grüneisen parameter (see Gruneisen's parameter for iron and Earth's core), L_T the adiabatic length scale, ΔS the entropy of crystallisation, $\Delta\rho$ the chemical density jump across the ICB (see Density distribution in the core) and G the gravitational constant. All these physical parameters can be estimated with more or less accuracy using combinations of geophysical data (mostly seismology) and mineral physics (see Core properties - Physical and Core properties - theoretical determination).

Convective mixing can be assumed to be sufficient to ensure a uniform mass rate $h(t)$ of radioactive heating in the core. Therefore, radioactive heating is not related to inner core growth and is simply equal to $M_N h(t)$, with M_N the mass of the core. This energy source can then be easily computed, provided one knows the concentration in radioactive elements in the core. Among all possible heat producing isotopes, ^{40}K has always been the most popular candidate, owing to its apparent depletion in the mantle compared Earth forming meteorites and its predicted metalization at high pressure that would allow it to enter the core. However, potassium is also somewhat volatile and its budget in the Earth is influenced by accretion processes. In addition, the concentration of potassium in the core depends strongly on the scenario of core formation, a process still far from perfectly understood (e.g. Stevenson, 1990). The most recent experiments devoted to the partitioning of potassium between iron and silicates (e.g. Gessmann and Wood, 2002; Rama Murthy et al., 2003; Lee and Jeanloz, 2003) favor a concentration of potassium in the core of O(100) ppm, producing less than $1\mathrm{TW}$ at present but exponentially more in the past. Such a value is too small to affect importantly the thermal evolution of the core (Labrosse, 2003) and radioactivity will not be considered further.

The gravitational energy actually comes in the equations as a compositional energy, due to a change of composition in a gradient of chemical potential (Braginsky and Roberts, 1995; Lister and Buffett, 1995). It is equal to the change of gravitational energy due only to chemical stratification of the core. Other sources of change of gravitational energy do not contribute significantly to this balance and are mostly stored as strain energy.

As can be seen on equations (2–4), for the different energy sources to be estimated, one needs to know the growth rate of the inner core. Since there is no direct way of measuring this number, one usually uses an estimate of the heat flow across the CMB to get this number from the energy balance (1). The heat flow across the CMB is not very well known (see Heat flow across the core mantle boundary) but for a value of $10\mathrm{TW}$ (say) and no radioactive elements, one gets approximately $Q_C = 5.5\mathrm{TW},\ Q_L = 2.8\mathrm{TW},\ E_G = 1.7$ (Labrosse, 2003).

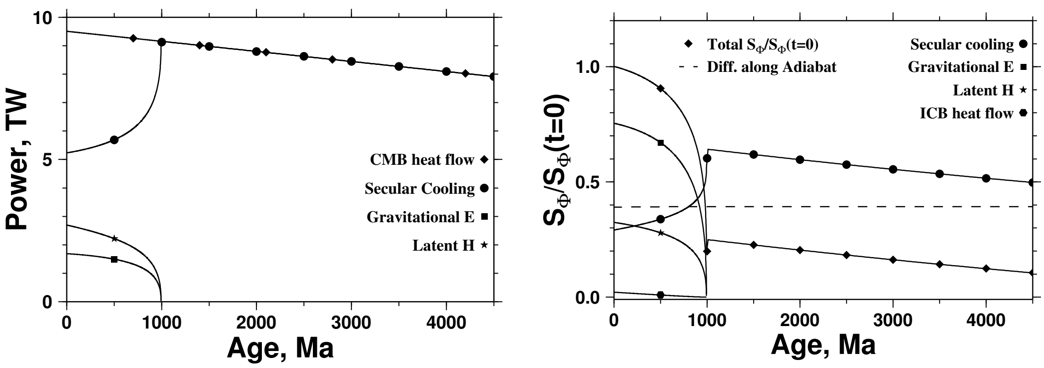

Figure 1: Energy (left) and entropy (right) balances of the core as a function of time for a typical evolution model without radioactivity. After Labrosse (2003)

The energy balance can be used for any time in the history of the Earth to give the growth history of the inner core and the evolution of all energy sources in the balance, providing the heat flow across the CMB is known as a function of time. Moreover, this equation can be integrated between the onset time of inner core crystallisation and the present to compute the age of the inner core (Labrosse et al., 2001). A typical example of time evolution of the energy balance is shown on figure 1 where the onset of inner core crystallisation, at an age of 1Ga, is marked by a qualitative change in the balance: before, only secular cooling is balancing Q_{CMB} (in absence of radioactivity) but this term is greatly decreased when the inner core starts to crystallise and both latent heat and compositional energy come in the balance.

Entropy equation

The energy equation does not involve the magnetic field directly and contains no contribution from the dissipative heating. This is a well known characteristic of convective engines: in contrary to Carnot engines, dissipation occurs inside the system and is then not lost. In order to relate the energy sources to the magnetic field generation, an entropy balance equation must be written. This equation comes from a combination of the momentum balance equation and the energy balance equation written above (e.g. Braginsky and Roberts, 1995) and states that the entropy that flows out through the CMB is balanced by the sum of the entropy that flows in due to the different heat sources and the entropy that is produced by non reversible processes, mostly ohmic dissipation and conduction along the adiabatic temperature profile. This equation does not directly involve the gravitational energy, since it is not a heat source. It can however be brought back into the equation by use of the energy balance to suppress the heat flow across the CMB, giving an efficiency equation,

$$\begin{aligned}\Phi + T_D \int_{OC} k \left(\frac{\nabla T}{T}\right)^2 dV = & \frac{T_D}{T_{CMB}} \left[E_G + \frac{T_{ICB} - T_{CMB}}{T_{ICB}} \left(Q_{ICB} + Q_L\right) \right. \\ & \left. + \frac{T_R - T_{CMB}}{T_R} Q_R + \frac{T_C - T_{CMB}}{T_C} Q_C \right], \end{aligned} \tag{5}$$

where it can be seen that each energy source contributes in maintaining both the total ohmic dissipation Φ and the conduction along the adiabatic gradient, but with different efficiency factors. In particular, this equation shows that the gravitational energy has an efficiency factor that is the ratio of the effective temperature T_D at which ohmic dissipation occurs to the temperature at the CMB, T_{CMB}, whereas all heat sources efficiency factors have a contribution from the classical Carnot engine efficiency factor, $(T_X - T_{CMB})/T_X$, with T_X the temperature at which the heat source X is provided. This shows that compositional energy is more efficiently transformed in dissipation than all heat sources and that the efficiency of each heat source depends on the temperature at which it is provided.

The energy sources on the right hand side of (5) are the same as that appearing in the energy balance (1) and, except for T_D, their efficiency factors can be expressed using the parameters characterising the reference state of the core. It can then be proved that all terms, except the radioactive heating one, is a function of the radius of the inner core and is proportional to its growth rate. This means that, if the heat flow across the CMB is known, this growth rate can be computed from the energy equation (1), and the entropy equation (5) then gives the ohmic dissipation that is maintained. Alternatively, one can take the opposite view and compute the growth rate of the inner core that is required to maintain a given ohmic dissipation, the energy balance being then used to get the heat flow across the CMB that makes this growth rate happen. Unfortunately, the ohmic dissipation in the core is no better known that the heat flow across the CMB, since it is dominated by small scales of the magnetic field and possibly by the invisible toroidal part of it (Gubbins and Roberts, 1987; Roberts et al., 2003). However, the value of $Q_{CMB} = 10\mathrm{TW}$ used above gives a contribution of ohmic dissipation to the entropy balance $\Phi/T_D = 500\mathrm{MWK}^{-1}$. The temperature T_D is not well known but is bounded by the temperature at the inner core boundary and the CMB and this gives $\Phi \simeq 2\mathrm{TW}$.

The evolution with time of the entropy balance associated with a given heat flow evolution can be computed and the example shown above gives the result of figure 1. An interesting feature is the sharp increase of the ohmic dissipation in the core when the inner core starts crystallising, latent heat and, even more so, gravitational energy being more efficient than secular cooling. Unfortunately, the link between this ohmic dissipation and the magnetic field observed at the surface of the Earth is far from obvious and the detection of such an increase in the paleomagnetic record is unlikely (Labrosse and Macouin, 2003).

Some alternative models for the average structure of the core involving some stratification have been proposed. In particular, the heat conducted along the adiabatic temperature gradient can be rather large (about 7 TW) and might be larger than the heat flow across the CMB (see Heat flow across the core mantle boundary). In this case, two different models have been proposed. In the first one, the adiabatic temperature profile is maintained by compositional convection against thermal stratification, except in a still very thin boundary layer, and this means that the entropy flow across the CMB is less than that required to maintain the conduction along the average temperature profile. In other words, the compositional convection has to fight against thermal stratification to maintain the adiabatic temperature profile in addition to maintaining the dynamo. In the second model (see Subadiabatic temperature gradients and Labrosse et al. (1997); Lister and Buffett (1998)), a sub-adiabatic layer of several hundreds of km is allowed to develop at the top of the core and the entropy flow out of the core balances the conduction along the average temperature gradient. In this case, the compositional energy is entirely used for the dynamo. Which of these two options would be chosen by the core is a dynamical question that cannot be addressed by simple thermodynamic arguments as used here.

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Cross references

Adiabatic gradient in the core
Boundary layers in the core
Convection, chemical
Core Composition
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Core properties - theoretical determination
Density distribution in the core
Gruneisen's parameter for iron and Earth's core
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